Exercise 1. *Verify the following:*

(*i*) *if* X *is a non-negative integer values r.v., then* $\mathbb{P}(X \geq 1) \geq \mathbb{E}(X)^2/\mathbb{E}(X^2)$,

(*ii*) *if* $X \sim \mathrm{Bi}(n, p)$*, then* $\mathbb{P}(X \geq k) \leq \binom{n}{k} p^k \leq \left(\frac{enp}{k}\right)^k$,

(*iii*) *if* $X \sim \mathrm{Hyp}(n, b, d)$*, then* $\mathbb{P}(X \geq k) \leq \binom{d}{k} \left(\frac{b}{n}\right)^k \leq \left(\frac{ebd}{kn}\right)^k$.

Exercise 2. *Suppose* $0 \leq p \leq p' \leq 1$ *and* $0 \leq M \leq M' \leq \binom{n}{2}$ *and* $\mathcal{P}$ *is an increasing graph property. Show that*

(*i*) $\mathbb{P}(G(n, p) \in \mathcal{P}) \leq \mathbb{P}(G(n, p) \in \mathcal{P})$ *and*

(*ii*) $\mathbb{P}(G(n, M) \in \mathcal{P}) \leq \mathbb{P}(G(n, M') \in \mathcal{P})$.

Exercise 3. *Suppose* $\mathcal{P}$ *is an increasing property, let* $M = M(n) \to \infty$*, and suppose* $\delta > 0$ *is a constant with* $(1+\delta)M/N = (1+\delta)M/\binom{n}{2} \leq 1$*. Set* $p = p(n) = M/N$*. Show that:*

(*i*) *If* $\mathbb{P}(G(n, p) \in \mathcal{P}) \to 1$*, then* $\mathbb{P}(G(n, M) \in \mathcal{P}) \to 1$.

(*ii*) *If* $\mathbb{P}(G(n, p) \in \mathcal{P}) \to 0$*, then* $\mathbb{P}(G(n, M) \in \mathcal{P}) \to 0$.

(*iii*) *If* $\mathbb{P}(G(n, M) \in \mathcal{P}) \to 1$*, then* $\mathbb{P}(G(n, (1+\delta)p) \in \mathcal{P}) \to 1$.

(*iv*) *If* $\mathbb{P}(G(n, M) \in \mathcal{P}) \to 0$*, then* $\mathbb{P}(G(n, (1-\delta)p) \in \mathcal{P}) \to 0$.

Exercise 4. *For every* $\eta > 0$ *there is* C *such that if* $d = pn \geq C$*, then* $G(n, p)$ *is a.s.* (p, η)*-uniform, i.e., for all* U, $W \subset V$*, with* $U \cap W = \emptyset$ *and* $|U|, |W| \geq \eta n$*, we have*

$$\left| e(U, W) - p|U||W| \right| \leq \eta p |U||W|.$$

Exercise 5. *For every* $0 < p = p(n) < 1$ *and* $d = np$*, the random graph* $G(n, p)$ *is a.s.* $(p, e^{3/2}\sqrt{d})$*-bijumbled, i.e., for all* U, $W \subset V$*, with* $U \cap W = \emptyset$ *and* $1 \leq |U| \leq |W| \leq pn|U|$*, we have*

$$\left| e(U, W) - p|U||W| \right| \leq e^{3/2}\sqrt{d}\sqrt{|U||W|}.$$

Why do we have the condition $1 \leq |U| \leq |W| \leq pn|U|$ *in the definition of bijumbled?*

Exercise 6. *Let* $G = G^n = (V, E)$ *be a* (p, α)*-bijumbled graph. Show that for all* $U \subset V$,

$$\left| e(G[U]) - p\binom{|U|}{2} \right| \leq \alpha |U|.$$

Exercise+ 7. *Prove that for any* $\eta > 0$ *and any* Δ*, there is* C *such that a.e.* $G = G(n, p)$ *with* $p = C/n$ *satisfies*

$$G(n, p) \to_\eta \mathcal{T},$$

where $\mathcal{T}$ *is the family of all trees* $T = T^t$ *with* $t \leq n/C$ *and* $\Delta(T) \leq \Delta$.
How about replacing trees by cycles ? (++)

Exercise+ 8. *Show*

(*i*) $p_0 = p_0(n) = n^{-2/3}$ *is the threshold for* $K^4 \subset G(n, p)$ *and it is coarse,*

(*ii*) $p_0 = n^{-2/3}$ *is the threshold for* $G(n, p) \to (K^3)^v_2$,

(*iii*) $p_0 = n^{-1/2}$ *is the threshold for* $G(n, p) \to (K^3)^e_2$

(*iv*) $p_0 = n^{-1/2}$ *is the threshold for* $G(n, p) \to (K^3)^e_r$ *for every* $r \geq 2$,

(*v*) $p_0 = n^{-1/2}$ *is the threshold for* $G(n, p) \to_{1/2+\eta} K^3$.

Exercise 9. *Show that every increasing property admits a threshold.*

Exercise+ 10. *Show that*

$$\mathbb{P}(\mathsf{G}(n,p) \text{ is connected}) = \begin{cases} 0 & \text{if } \lim_n c_n = -\infty, \\ e^{-e^{-c}} & \text{if } \lim_n c_n = c \in \mathbb{R}, \\ 1 & \text{if } \lim_n c_n = \infty. \end{cases}$$

if $p = \frac{1}{n}(\log n + c_n)$.