

Exercise 1. Among all the n -vertex p -chromatic graphs $T_{n,p}$ is the (only) graph maximizing $e(T_{n,p})$.

Exercise 2. If $\chi(G_n) = p$ and $e(G_n) = e(T_{n,p}) - s$ then in a p -colouring of G_n , the size of the i -th colour-class n_i satisfies

$$\left| n_i - \frac{n}{p} \right| < c\sqrt{s+1}.$$

Exercise 3. (i) Let \mathcal{C} denote the family of all cycles. Determine $\text{ex}(n, \mathcal{C})$.

(ii) Determine $\text{ex}(n, P_3)$.

(iii) Prove that if minimum degree of (G_n) is at least k then G_n contains all trees on $k+1$ vertices.

(iv) Prove that for any tree T_k , $\text{ex}(n, T_k) \leq (k-2)n$.

Exercise 4. (i) Verify $\frac{(n-p+1)^p}{p!} \leq \binom{n}{p} \leq \frac{n^p}{p!}$. When are those estimates sharp?

(ii) For $a \gg c \gg b$, estimate $\frac{\binom{a-b}{c-b}}{\binom{a}{c}}$.

Exercise 5. Let \mathcal{L} be a finite family of graphs. Show that $\text{ex}(n, \mathcal{L}) = O(n)$ iff \mathcal{L} contains a forest. Can the finiteness be omitted?

Exercise 6. Prove that among 5 points there are always 4 in a convex position.

Exercise 7. Prove Ramsey's theorem for graphs, i.e., that $R(k, \ell) \leq \binom{k+\ell-2}{k-1}$.

Exercise 8. Prove the Erdős-Szekeres-theorem using only Ramsey's theorem for 3-uniform hypergraphs.

Exercise 9. Prove (with details) the following result of Szemerédi: if G_n contains no K_4 and $\alpha(G_n) = o(n)$, then $e(G_n) = n^2/8 + o(n^2)$.